This is a closed book, closed note test, and should be your work only. You may use a calculator during this exam. Stay calm, read all the instructions, show your work and write your answers on this test. Ask the instructor for clarification if you are confused as to what is being asked.

1. How many strings of eight lowercase English letters are there
   a. that contain no vowels, if letters can be repeated?
   b. that start with a vowel, if letters cannot be repeated?

2. How many numbers must be selected from the set \{1, 4, 5, 8, 10, 13, 14, 17\} to guarantee that at least one pair of these numbers add up to 18?

3. How many ways are there to distribute three balls into eight boxes if
   a. the balls are labeled, but the boxes are unlabeled?
   b. the balls are unlabeled, but the boxes are labeled?
4. How many ways can a set of two positive integers less than 100 be chosen?

5. What is the probability of the following events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

   a. *a* is the first letter of the permutation and *z* is the last letter.

   b. *a* and *z* are next to each other in the permutation.

6. Suppose that 8% of the patients tested in a clinic are infected with HIV. Furthermore, suppose that when a blood test for HIV is given, 98% of the patients infected with HIV test positive and that 3% of the patients not infected with HIV test positive. What is the probability that

   a. a patient testing positive for HIV with this test is infected with it?

   b. a patient testing negative for HIV with this test is infected with it?
7. What is the probability that the sum of the numbers on two dice is even when they are rolled?

8. How many ways are there to assign three different jobs to five employees if each employee can be given no more than one job?

9. A coin is biased so that the probability a head comes up when it is flipped is 0.6. What is the expected number of heads that come up when it is flipped 4 times?

10. How many one-to-one functions are there from a set with five elements to a set with 7 elements?
11. What is the probability of the following events when we randomly select a permutation of \( \{1, 2, 3, 4\} \)?

a. 1 precedes 4.

b. 4 precedes 1 and 4 precedes 2.

c. 4 precedes 3 and 2 precedes 1.

12. Use the Binomial Theorem to show the following:

\[
\sum_{k=0}^{n} C(n, k) = 2^n.
\]

BONUS QUESTION (in the style of Math Jeopardy): This Swiss mathematician is best known for his posthumous work that described the known results in probability theory and in enumeration including the application of probability theory to games of chance and his introduction of the theorem known as the law of large numbers.