Your test will be of similar content and material, but the questions will be different and the questions may be asked in a different manner. The test questions will come from all the material we have covered in class.

This is a closed book, closed note test, and should be your work only. You may use a calculator during this exam. Stay calm, read all the instructions, show your work and write your answers on this test. Ask the instructor for clarification if you are confused as to what is being asked.

1. Suppose that \( a_0, a_1, a_2, a_3, \ldots \) is a sequence defined by the recurrence relation given by
   \[
   a_n = 2a_{n-1} + 4n, \quad n \geq 1, \quad a_0 = 1.
   \]
   a. Calculate \( a_1, a_2, a_3, \) and \( a_4. \)

   \[
   a_1 = \\
   a_2 = \\
   a_3 = \\
   a_4 = 
   \]

   b. Find an explicit formula for the sequence. You need not prove your formula is correct by induction, but you should simplify your answer as much as possible.
2a. Find an explicit formula for the following recurrence relation with given initial conditions.

\[ a_n = 6a_{n-1} - 9a_{n-2}, \quad n \geq 2, \quad a_0 = 1 \quad a_1 = \_ \_ \_ \]

b. **Prove** that your formula is correct using induction.
3. Find an explicit formula for the following recurrence relation with given initial conditions. You need **not** prove your formula is correct by induction, but you should simplify your answer as much as possible.

\[ a_n = -8a_{n-1} - 15a_{n-2}, \quad n \geq 2, \quad a_0 = 2, \quad a_1 = 2. \]

4. There are 323 farms in Monmouth County that have at least one of horses, cows, and sheep. If 224 have horses, 85 have cows, 57 have sheep, and 18 farms have all three types of animals, how many farms have exactly two of these three types of animals?
5. Use generating functions to solve the following recurrence relation:

\[ a_n = 10a_{n-1} - 25a_{n-2} \quad n \geq 2, \quad a_0 = 1, \quad a_1 = 5. \]

6. Find the general form of the solutions of a linear homogeneous recurrence relation if its characteristic equation has the roots -1, -1, -1, 2, -5, -5.
7. Prove that $f_{4n}$ is divisible by three where $f_j$ is the Fibonacci sequence with $f_j = f_{j-1} + f_{j-2}$ and $f_0 = 0$ and $f_1 = 1$.

8. Find a closed form for the generating function for the sequence $1, 1, 1, 1, 2, 3, 4, \ldots$