MATH 4473, Discrete Mathematical Structures
Practice Test 3, Spring 2010

Your test will be of similar content and material, but the questions will be different and the questions may be asked in a different manner. The test questions will come from all the material we have covered in class.

This is a closed book, closed note test, and should be your work only. You may use a calculator during this exam. Stay calm, read all the instructions, show your work and write your answers on this test. Ask the instructor for clarification if you are confused as to what is being asked.

1. Use the following graph to answer the following questions.

![Graph Image]

a. Using appropriate graph terminology, describe what type of graph this is. 
   
   **undirected multigraph**

b. Find the incidence matrix to represent the graph above.

   \[
   V_1 \begin{bmatrix}
   a & b & c & d & e \\
   1 & 0 & 0 & 0 & 0 \\
   1 & 0 & 0 & 0 & 0 \\
   0 & 0 & 1 & 1 & 1 \\
   1 & 0 & 0 & 1 & 1 
   \end{bmatrix}
   \]

2. Determine if the following pair of graphs is isomorphic. If they are isomorphic, write down the isomorphism. If they are not isomorphic, explain why they are not.

   ![Graphs V and U]

   These two graphs are not isomorphic. Graph V has a vertex with degree 4 while none of the vertices of graph U have degree 4.
3. Use the graph to answer the following questions.

![Graph Image]

a. Write down the adjacency list for this graph.

<table>
<thead>
<tr>
<th>V₁</th>
<th>V₂</th>
<th>V₃</th>
<th>V₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>V₂</td>
<td>V₁</td>
<td>V₃</td>
<td></td>
</tr>
<tr>
<td>V₃</td>
<td>V₄</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V₄</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. What is the outdegree of V₂?

2

c. What is the indegree of V₂?

0

d. Write down a simple path from V₂ to V₄ of length 3.

Using edges: a, c, d

Using vertices: V₂, V₃, V₄

e. Write down the adjacency matrix for this graph.

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
= A
\]

f. Find the matrix that gives the number of paths of length 3 for this graph. You need not show your work.

Using Calculator.

\[
A^3 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
4. Use the following graph to answer the following questions.

[Diagram of a graph]

a. Determine if the graph has an Euler Path. If it does, write it down. If it doesn't, explain why. **Using edges:** \( e, g, f, c, d, e, b, a \). **Using vertices:** \( F, E, D, B, D, C, B, A \). Other correct answers but start/end must be at A/F.

b. Determine if the graph has an Euler Circuit. If it does, write it down. If it doesn't, explain why. **There is no Euler Circuit because A and F have degree 1.**

c. Determine if the graph has a Hamiltonian Path. If it does, write it down. If it doesn't, explain why. **Using edges:** \( e, f, c, b, a \). **Using vertices:** \( F, E, D, C, B, A \). Other correct answers.

d. Determine if the graph has a Hamiltonian Circuit. If it does, write it down. If it doesn't, explain why. **There is no Hamiltonian Circuit. There is no way to complete the loop from A to F (or vice versa).**

5. Find the chromatic number of the graph below. Indicate how you would color the vertices.

[Diagram of a graph with colors]

**Chromatic Number = 3**
6. Suppose that a connected planar graph has six vertices, each of degree four. Into how many regions is the plane divided by a planar representation of this graph?

\[ R = e - V + 2 = 12 - 6 + 2 = 8 \text{ regions.} \]

7a. Draw the graph \(K_5\).

7b. Draw the graph of \(K_{3,4}\).

7c. Which one of the above two graphs is bipartite? Explain.

\(K_{3,4}\) is bipartite. You can color the vertices along the top row one color and the vertices along the bottom row another color. \(K_5\) needs at least 3 colors so is not bipartite.

8. Construct the dual graph for the given map.
9. Determine whether the given graph is planar. If so, draw it so that no edges cross.

10. In a round-robin tournament the Tigers beat the Blue Jays, the Tigers beat the Cardinals, the Tigers beat the Orioles, the Blue Jays beat the Cardinals, the Blue Jays beat the Orioles, and the Cardinals beat the Orioles. Model this outcome with a directed graph.

11. The mathematics department has six committees each meeting once a month. The committees are $C_1 = \{\text{Armey, Brand, Zink}\}$, $C_2 = \{\text{Brand, Lee, Rosen}\}$, $C_3 = \{\text{Armey, Rosen, Zink}\}$, $C_4 = \{\text{Lee, Rosen, Zink}\}$, $C_5 = \{\text{Armey, Brand}\}$, and $C_6 = \{\text{Brand, Rosen, Zink}\}$.

   a. Represent this situation with the appropriate graph.

   b. Using your graph, how many different meeting times must be used to ensure that no member is scheduled to attend two meetings at the same time?

\[5 \text{ times} \quad (C_4 \text{ and } C_5 \text{ can meet at same time})\]