

Introduction to Quantum Mechanics

Physics 3043

Practice Test for the material from Chapter 1 and Chapter 2 sections 1-3

(your test will not have this many questions/problems)

1. List the properties of ψ it must have in order that ψ^* be a proper solution to the Schrödinger Equation, viz. That ψ^* be proportional to the probability.
2. Starting with the time-dependent Schrödinger Equation in one dimension, show how the time independent equation is obtained by the separation of variables, i.e. $\psi(x,t) = \psi(x)\phi(t)$.
3. Given that
$$\psi(x) = \begin{cases} A(2-x^2) & -\sqrt{2} \leq x \leq \sqrt{2} \\ 0 & \text{Otherwise} \end{cases}$$

Determine the value of A that will normalize $\psi(x)$.

4. For the wavefunction given in problem 3 find $\langle x \rangle$ and $\langle x^2 \rangle$
5. For the wavefunction given in problem 3 find $\langle p \rangle$ and $\langle p^2 \rangle$

6. Show that $[x,p] = i\hbar$ by using the operators for x and p on a function f.
Note: $[a,b] = ab-ba$

7. If the ladder operators are defined by

$$a_{\pm} \equiv \frac{1}{\sqrt{2m}} \left(\frac{\hbar}{i} \frac{d}{dx} \pm im\omega x \right)$$

show that

$$a_+ a_- f = \left(\frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} m \omega^2 x^2 \right) f - \frac{\hbar\omega}{2} f$$

8. If ψ_1 and ψ_2 are solutions of the Schrödinger Equation show that a linear combination is also a solution, ie. $\psi = a\psi_1 + b\psi_2$ is also a solution.

9. List the Heisenberg Uncertainty Principle for position and momentum. From problems 4 and 5 describe how the uncertainty product: $\Delta x \Delta p$ would be calculated.

10. If ψ_1 is a solution to the Schrödinger equation with an energy E_1 and ψ_2 is a solution to the Schrödinger equation with an energy, E_2 , what is the integral of $\psi_1^* \psi_2$ over the entire domain? Assume these are normalized and correspond to the same Hamiltonian.

Review

Need To Know:

Schrödinger equation in one dimension (both time dependent and time independent)

position operator

linear momentum operator

kinetic energy operator

potential energy operator

Hamiltonian operator in one dimension

how to calculate the standard deviation

x

$\langle f(x) \rangle$

$\langle g(p) \rangle$

what a boundary condition is and how to apply

what an initial condition is

mn

general solution to infinite square well

how to solve a differential equation by an infinite series

what a ladder operator does on a wavefunction

how to obtain the complex conjugate

how to use a recursion relation

how to normalize a wavefunction

what probability density is

Taylor series expansion of e^x , $\cos(x)$, $\sin(x)$

Euler relation: $e^{i\theta} = \cos(\theta) + i \sin(\theta)$

the definition of an even function

the definition of an odd function

the integration of even and odd functions over anti-symmetric limits