PHYS 2015
Test #4
Spring 2009 (Buckley)

Either circle the letter corresponding to the best answer or fill in the missing information in each of the following questions. Question 1 is worth five points and 2-9 are each worth two points.

1. Use the capitalized letter on the left to match each translational symbol with its angular analog on the right.
   
   A. a
   B. x
   C. v
   D. m
   E. p

   \[ \begin{align*}
   &\text{B} & \theta \\
   &\text{E} & \omega \\
   &\text{D} & \alpha \\
   \end{align*} \]

2. A sphere is rolling along a surface with a rotational kinetic energy of 15 J. If the angular velocity of the sphere is increased by a factor of 4, the kinetic energy will
   A. increase to 30 J
   B. increase to 60 J
   C. increase to 120 J
   D. increase to 240 J
   E. not change

3. If a constant net torque is applied to an object, the object will:
   A. rotate with constant angular velocity.
   B. rotate with constant angular acceleration.
   C. have an increased moment of inertia.
   D. have a decreased moment of inertia.

4. A spinning figure skater is rotating at 3.50 rad/s with a moment of inertia of 2.50 kg·m². If the arms are pulled in so the moment of inertia of the skater becomes 1.28 kg·m², what is the skater’s final angular speed?
   A. 7.00 rad/s
   B. 0.88 rad/s
   C. 1.75 rad/s
   D. 14.0 rad/s
Consider the following situation as you answer questions 5 – 9.

A boy and a girl are riding on a merry-go-round which is turning at a constant rate. The boy is near the outer edge and the girl is closer to the center.

5. Who has the greater angular displacement in the same time period?
   A. the boy
   B. the girl
   C. Both have the same non-zero angular displacement.
   D. Both have zero angular displacement.

6. Who has the greater angular speed?
   A. the boy
   B. the girl
   C. Both have the same non-zero angular speed.
   D. Both have zero angular speed.

7. Who has the greater linear speed?
   A. the boy
   B. the girl
   C. Both have the same non-zero linear speed.
   D. Both have zero linear speed.

8. Who has the greater centripetal acceleration?
   A. the boy
   B. the girl
   C. Both have the same non-zero centripetal acceleration
   D. Both have zero centripetal acceleration.

9. Who has the greater tangential acceleration?
   A. the boy
   B. the girl
   C. Both have the same non-zero tangential acceleration.
   D. Both have zero tangential acceleration.

10. As one looks at a spinning wheel from the top it is spinning clockwise. The direction of the angular velocity of the wheel is:

    A. tangent to the edge of the wheel pointing in the direction of motion.
    B. tangent to the edge of the wheel pointing opposite to the direction of motion.
    C. pointing toward the center of the wheel
    D. pointing directly up perpendicular to the center of the wheel.
    E. pointing directly down perpendicular to the center of the wheel.
Problems. BE SURE TO SHOW YOUR WORK TO RECEIVE CREDIT. IF YOU USE SCRATCH PAPER, PLEASE PUT YOUR NAME ON IT AND FOLD IT UP AND TUCK IT INSIDE YOUR TEST WHEN YOU TURN IT IN.

11. (10 points) The blades of a fan running at low speed turn at 200 rpm. When switched to high speed, the speed of the blades increases uniformly to 300 rpm in 4.5 s.

A. What is the magnitude of the angular acceleration of the blades during this 4.5 s in units of rad/s²?

$$\omega_0 = 200 \text{ rpm} \times \frac{2\pi \text{ rad}}{1 \text{ rev.}} \times \frac{1 \text{ min}}{60 \text{ s}} = \frac{20.7 \text{ rad}}{s}$$

$$\omega = 300 \text{ rpm} \times \frac{2\pi \text{ rad}}{1 \text{ rev.}} \times \frac{1 \text{ min}}{60 \text{ s}} = \frac{31.4 \text{ rad}}{s}$$

$$t = 4.5 \text{ s}$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{31.4 \text{ rad/s} - 20.7 \text{ rad/s}}{4.5 \text{ s}} = \frac{2.74 \text{ rad/s}^2}{\text{s}}$$

B. How many revolutions have the blades gone through in 3.0 s after starting the acceleration from 200 rpm?

$$\omega = \omega_0 + \frac{1}{2} \alpha t^2 = 20.7 \text{ rad/s} + \frac{1}{2} \left(2.74 \text{ rad/s}^2\right)(3.0 \text{ s})^2$$

$$= 73.2 \text{ rad/s} \approx 11.7 \text{ revolutions}$$

C. If the radius of the fan blades is 25.0 cm, what is the tangential acceleration of a point on the outside edge of a blade 3.0 s after starting the acceleration from 200 rpm?

$$a_{tan} = r \alpha = (0.25 \text{ m}) \times \frac{2.74 \text{ rad}}{s} = 0.58 \text{ m/s}^2$$

D. If the radius of the fan blades is 25.0 cm, what is the centripetal acceleration of a point on the outside edge of a blade 3.0 s after starting the acceleration from 200 rpm?

$$a_{cm} = \omega^2 r = \left(20.7 \frac{\text{rad}}{s}\right)^2 (0.25 \text{ m}) = (195 \frac{\text{m}}{s^2})$$

$$\omega = \omega_0 + \alpha t = 20.7 \frac{\text{rad}}{s} + \left(2.74 \frac{\text{rad}}{s^2}\right)(3.0 \text{ s}) = 27.9 \frac{\text{rad}}{s}$$

E. If the radius of the fan blades is 25.0 cm, what is the magnitude of the total linear acceleration of a point on the outside edge of a blade 3.0 s after starting the acceleration from 200 rpm?

$$a = \sqrt{a_{tan}^2 + a_{cm}^2} = \sqrt{0.58^2 + 195^2} \approx 195 \frac{\text{m}}{s^2}$$
12. (10 points) Ok, I messed up this problem in class yesterday so let’s work it out now. The two trees in the figure are 8.5 m apart. A backpacker is trying to lift his 15.0 kg pack out of the reach of bears. The goal here was to find the force, F, the backpacker had to exert to lift the backpack to some predetermined sag in the rope.

A. If the rope is to sag 1.5 m at the middle, what is the angle between the rope and the horizontal direction?

\[ \theta = \tan^{-1} \left( \frac{1.5}{8.5} \right) = 19.4^\circ \]

B. What is the total downward force exerted by the backpack?

\[ W_g = 147 \text{ N} \]

C. Considering that each half of the rope has to exert one-half of the upward force required to match the downward force, how much vertical force must each half of the rope exert?

\[ \frac{147 \text{ N}}{2} = 73.5 \text{ N contributed by each half} \]

D. What is the magnitude of the tension in the rope (same as the F required to be applied to the rope) required to exert the force indicated in part C?

\[ F = \frac{73.5}{\sin 19.4^\circ} = 221 \text{ N} \]

E. Would it be possible to exert sufficient force for there to be no sag in the rope? If so, determine the force. If not, explain using forces why not. Use the back of the page if necessary. NO - THERE MUST BE SOME UPWARD FORCE TO COUNTERACT THE WEIGHT. IF ONE USED A PERFECTLY RIGID ROD IT COULD PROVIDE SUCH AN UPWARD FORCE DUE TO ITS MODULUS.
13. (10 points) Calculate:

A. the tension $F_T$ in the wire that supports the 27-kg beam shown in the figure.

$x$-direction: $F_W \cos \theta - F_T \cos 40^\circ = 0$

$y$-direction: $F_W \sin \theta + F_T \sin 40^\circ - mg = 0$

T: Take $\theta = 40^\circ$

$x$-direction: $-F_W \sin \theta + F_T \sin 40^\circ = 0$

Look at $y$: $\frac{\theta}{2}$ cancels: $-F_W \sin \theta + F_T \sin 40^\circ = 0$

$\Rightarrow F_W \sin \theta = F_T \sin 40^\circ$

Sub into $y$-dir: $F_W \sin \theta + F_T \sin 40^\circ - mg = 0$

$F_T \sin 40^\circ = mg$

$F_T = \frac{mg}{\sin 40^\circ} = \frac{(27 \text{ kg})(9.8 \text{ m/s}^2)}{2 \sin 40^\circ} = 205.8 \text{ N}$

B. the force $F_W$ exerted by the wall on the beam (give magnitude and direction).

From above $F_T = 205.8 \text{ N}$

From the $x$-direction: $F_W \cos \theta - 205.8 \cos 40^\circ = 0$

$F_W \cos \theta = 205.8 \cos 40^\circ 

From the $y$-direction: $F_W \sin \theta + F_T \sin 40^\circ - mg = 0$

$F_W \sin \theta = -F_T \sin 40^\circ + mg = 132 \text{ N}$

$\tan \theta = \frac{132.3}{157.7} \approx 0.84$

$\Rightarrow \theta = \tan^{-1} 0.84 = 40^\circ$

Magn: $\frac{157.7}{\cos 40^\circ} = 205.8 \text{ N}$

Turn in your index card with the test.