PHYS 1115
Quiz #4
Spring 2009 (Buckley)

Show your work on all numerical problems to receive full credit.

1. 5 points) A wheel 40-cm in diameter accelerates uniformly from 240 rpm to 360 rpm in 7.5 s.

   a. What is its angular acceleration?

   \[ \omega = \omega_0 + \alpha t \]
   \[ \alpha = \frac{\omega - \omega_0}{t} \]
   \[ \omega = 240 \text{ rpm} \times \frac{2\pi \text{ rad}}{1 \text{ rpm}} \times \frac{1 \text{ min}}{60 \text{ s}} = 25.1 \text{ rad/s} \]
   \[ \omega = 360 \text{ rpm} \times \frac{2\pi \text{ rad}}{1 \text{ rpm}} \times \frac{1 \text{ min}}{60 \text{ s}} = 37.7 \text{ rad/s} \]
   \[ \alpha = \frac{37.7 - 25.1}{7.5} \text{ rad/s}^2 \]
   \[ \alpha = 1.68 \text{ rad/s}^2 \]

   OR

   \[ \alpha = \frac{16}{7.5} \text{ rpm} \times \frac{2\pi \text{ rad}}{1 \text{ rpm}} \times \frac{1 \text{ min}}{60 \text{ s}} \]

   b. How many revolutions does the wheel make in this 7.5 s?

   \[ \omega = 25.1 \text{ rad/s} \]
   \[ \omega = 37.7 \text{ rad/s} \]
   \[ \alpha = 16.8 \text{ rad/s}^2 \]
   \[ \theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(37.7)^2 - (25.1)^2}{2(16.8)} \text{ rad} \]
   \[ \theta = 235.5 \text{ rad} \]
   \[ \theta = 37.5 \text{ revolutions} \]

   c. What distance does a point on the edge of the wheel travel in this time?

   \[ l = r\theta = (0.40 \text{ m}) \times (235.5 \text{ rad}) = 94.2 \text{ m} \]

2. 5 points) Our Sun currently has a radius of \( 7.0 \times 10^5 \) km and rotates about its axis with an angular velocity of 57 rad/day. As it progresses through its life cycle, it is expected to shrink to a white dwarf with a radius of about \( 7.0 \times 10^3 \) km. The moment of inertia of a sphere is \( \frac{2}{5} MR^2 \). What will the Sun's new angular velocity be when it shrinks to a white dwarf? Assume no mass is lost in the compression.

   \[ L_{\text{initial}} = L_{\text{final}} \]
   \[ I_{\text{initial}} \omega_{\text{initial}} = I_{\text{final}} \omega_{\text{final}} \]
   \[ \omega_{\text{final}} = \frac{I_{\text{initial}} \omega_{\text{initial}}}{I_{\text{final}}} \]
   \[ \omega_{\text{initial}} = \frac{2}{5} \frac{M R_e^2}{I_{\text{final}}} \]
   \[ \omega_{\text{final}} = \frac{R_{\text{initial}}^2}{R_{\text{final}}^2} \omega_{\text{initial}} = \left( \frac{7 \times 10^5 \text{ km}}{7 \times 10^3 \text{ km}} \right)^2 57 \text{ rad/day} \]
   \[ = 570000 \text{ rad/day} = 23700 \text{ rad/hr} = 6.6 \text{ rad/s} \]
3. (6 points) Consider the following system as you answer the questions. The weight of the stick is 2.0-kg and the center of gravity is indicated by c.g. on the diagram.

a. Find the clockwise torque (will have negative algebraic sign) about the pivot point indicated by the triangle.

   \[ \text{Clockwise torques:} \quad 45.0 \text{ N} \cdot 1.5 \text{ m} = -67.5 \text{ m N} \]
   \[ 55.0 \text{ N} \cdot 0.5 \text{ m} = -27.5 \text{ m N} \]
   \[ -95.0 \text{ m N} \]

b. Find the counterclockwise torque (will have positive algebraic sign) about the pivot point indicated by the triangle.

   \[ 25.0 \text{ N} \cdot 2.0 \text{ m} = 50.0 \text{ m N} \]
   \[ 2.0 \text{ kg} \times 9.8 \text{ m/s}^2 \times 0.5 \text{ m} = 9.8 \text{ m N} \]
   \[ 59.8 \text{ m N} \]

c. What is the overall torque about the pivot point and in which way will the stick accelerate—clockwise or counterclockwise?

   \[ \tau_{net} = 59.8 \text{ m N} + (-95.0 \text{ m N}) = -35.2 \text{ m N} \]

   Clockwise

4. (4 points) Answer the following short questions.

a. In the demonstration in class, I spun the bicycle wheel horizontally, stood on the rotating platform, and turned the wheel upside-down. Circle the answer that best describes what happened.

   i. The rotating platform began rotating in the direction the bicycle wheel was originally rotating.
   ii. The rotating platform began rotating in the direction opposite to that the bicycle wheel was originally rotating.
   iii. The platform did not rotate.

b. The demonstration mentioned in part a illustrates:

   i. the conservation of kinetic energy
   ii. the conservation of angular velocity
   iii. the conservation of angular momentum
   iv. the conservation of angular acceleration

   \[ \boxed{\text{iii}} \]
Translational and Rotational Analogues — these all refer to magnitudes. Remember many of these are vector quantities.

Based on the table on p. 217 (Algebra-based) and p. 275 (Calculus-based)

<table>
<thead>
<tr>
<th></th>
<th>Translation</th>
<th>Rotation</th>
<th>Connection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>$x$</td>
<td>$\theta$</td>
<td>$x=r\theta$ (introduced as $\theta=r\theta$)</td>
</tr>
<tr>
<td>Velocity</td>
<td>$v$</td>
<td>$\omega$</td>
<td>$v=r\omega$</td>
</tr>
<tr>
<td>Average</td>
<td>$v = \frac{\Delta x}{\Delta t}$</td>
<td>$\omega = \frac{\Delta \theta}{\Delta t}$</td>
<td></td>
</tr>
<tr>
<td>Instantaneous (Algebra)</td>
<td>$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$</td>
<td>$\omega = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$</td>
<td></td>
</tr>
<tr>
<td>Instantaneous (Calculus)</td>
<td>$v = \frac{dx}{dt}$</td>
<td>$\omega = \frac{d\theta}{dt}$</td>
<td></td>
</tr>
<tr>
<td>Acceleration</td>
<td>$a$</td>
<td>$\alpha$</td>
<td>$a=\theta^2$</td>
</tr>
<tr>
<td>Average</td>
<td>$a = \frac{\Delta v}{\Delta t}$</td>
<td>$\alpha = \frac{\Delta \alpha}{\Delta t}$</td>
<td></td>
</tr>
<tr>
<td>Instantaneous (Algebra)</td>
<td>$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$</td>
<td>$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \alpha}{\Delta t}$</td>
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<tr>
<td>Instantaneous (Calculus)</td>
<td>$a = \frac{dv}{dt}$</td>
<td>$\omega = \frac{d\theta}{dt}$</td>
<td></td>
</tr>
<tr>
<td>m, mass</td>
<td></td>
<td>l, moment of inertia</td>
<td>$I = \sum mr^2$</td>
</tr>
<tr>
<td>Kinetic Energy</td>
<td>$\frac{1}{2}mv^2$</td>
<td>$\frac{1}{2}I\omega^2$</td>
<td></td>
</tr>
<tr>
<td>Momentum</td>
<td>$p=mv$</td>
<td>$l=l\omega$</td>
<td></td>
</tr>
<tr>
<td>Work</td>
<td>$F \cdot d$</td>
<td>$\tau \theta$</td>
<td></td>
</tr>
<tr>
<td>Newton’s 2nd Law</td>
<td>$\sum F = ma$</td>
<td>$\sum \tau = I \alpha$</td>
<td></td>
</tr>
<tr>
<td>Kinematic Equations (constant acceleration, a or $\alpha$):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>v = $v_0 + at$</td>
<td>$\omega = \omega_0 + \alpha t$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x = $v_0 t + \frac{1}{2} at^2$</td>
<td>$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v^2 = v_0^2 + 2ax$</td>
<td>$\omega^2 = \omega_0^2 + 2\alpha \theta$</td>
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<td></td>
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<tr>
<td>$v = \frac{v + v_0}{2}$</td>
<td>$\omega = \frac{\omega + \omega_0}{2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

GSB Modified 4/15/09